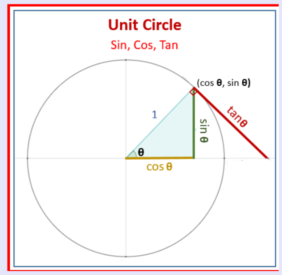
Trigonometry

Lecture 22



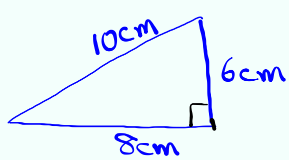
Feb 19-8:47 AM

Class Quiz 6

Use Heron's Formula to find the area of the triangle below

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \dfrac{a+b+c}{2}$



$s = \dfrac{6+8+10}{2} = \dfrac{24}{2} = 12$

$\text{Area} = \sqrt{12(12-6)(12-8)(12-10)}$

$= \sqrt{12 \cdot 6 \cdot 4 \cdot 2}$

$= \sqrt{576} = \boxed{24\ cm^2}$

Right Triangle

$6^2 + 8^2 = 10^2$ ✓

$\text{Area} \ \dfrac{bh}{2} = \dfrac{6 \cdot 8}{2} = \dfrac{48}{2} = \boxed{24\ cm^2}$

$P = a + b + c = 6 + 8 + 10 = \boxed{24\ cm}$

$\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{\cancel{2}} \cdot \overset{3}{\cancel{6}} \cdot 8 \cdot \overset{1}{\cancel{\sin 90^\circ}} = \boxed{24\ cm^2}$

Oct 3-11:35 AM

Verify

$$\frac{\csc x - \cot x}{\sec x - 1} = \cot x$$

$$\frac{\csc x - \cot x}{\sec x - 1} = \frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{\frac{1}{\cos x} - 1} = \frac{\cancel{\sin x}\cos x \cdot \frac{1}{\cancel{\sin x}} - \cancel{\sin x}\cos x \cdot \frac{\cos x}{\cancel{\sin x}}}{\sin x\cancel{\cos x} \cdot \frac{1}{\cancel{\cos x}} - \sin x\cos x}$$

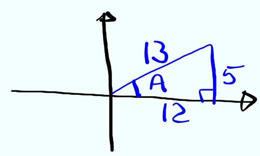
LCD = $\sin x \cos x$

$$= \frac{\cos x - \cos^2 x}{\sin x - \sin x \cos x}$$

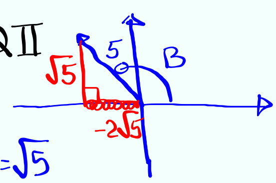
$$= \frac{\cos x\cancel{(1 - \cos x)}}{\sin x\cancel{(1 - \cos x)}} = \frac{\cos x}{\sin x} = \boxed{\cot x}$$

Oct 7-10:32 AM

$\sin A = \frac{5}{13}$, A is in QI.



$\cos B = \frac{-2\sqrt{5}}{5}$, B is in QII



$y^2 + (-2\sqrt{5})^2 = 5^2 \quad y^2 + 20 = 25$

$y^2 = 5 \quad y = \sqrt{5}$

$\sin(A + B)$

$= \sin A \cos B + \cos A \sin B = \frac{5}{13} \cdot \frac{-2\sqrt{5}}{5} + \frac{12}{13} \cdot \frac{\sqrt{5}}{5}$

$= \frac{-10\sqrt{5} + 12\sqrt{5}}{65} = \boxed{\frac{2\sqrt{5}}{65}}$

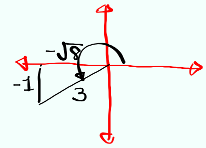
$\cos(A - B)$

$= \cos A \cos B + \sin A \sin B$

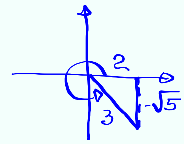
$= \frac{12}{13} \cdot \frac{-2\sqrt{5}}{5} + \frac{5}{13} \cdot \frac{\sqrt{5}}{5} = \boxed{\frac{-19\sqrt{5}}{65}}$

Oct 7-10:39 AM

$\sin A = \frac{-1}{3}$, A is in QIII



$\cos B = \frac{2}{3}$, B is in QIV



Find $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$= \frac{\frac{1}{\sqrt8} + \frac{-\sqrt5}{2}}{1 - \frac{1}{\sqrt8}\cdot\frac{-\sqrt5}{2}} = \frac{2-\sqrt{40}}{2\sqrt8+\sqrt5}$$

LCD $= 2\sqrt8$

$\sqrt4\sqrt2$ $\quad 2\sqrt{64}\sqrt5$ $\quad \sqrt{100}\sqrt2$

Rationalize the deno.

$$\frac{2-\sqrt{40}}{2\sqrt8+\sqrt5}\cdot\frac{2\sqrt8-\sqrt5}{2\sqrt8-\sqrt5} = \frac{4\sqrt8 - 2\sqrt5 - 2\sqrt{320} + \sqrt{200}}{4\sqrt{64} - 2\cancel{\sqrt{40}} + \cancel{2\sqrt{40}} - \sqrt{25}}$$

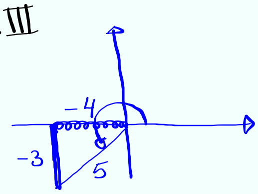
$$= \frac{8\sqrt2 - 2\sqrt5 - 16\sqrt5 + 10\sqrt2}{4\cdot8 - 5} = \frac{18\sqrt2 - 18\sqrt5}{32-5} = \frac{\overset{2}{\cancel{18}}(\sqrt2-\sqrt5)}{\underset{3}{\cancel{27}}}$$

$$= \frac{2(\sqrt2-\sqrt5)}{3}$$

Oct 7-10:47 AM

$\sin x = \frac{-3}{5}$ $\quad x$ is in QIII

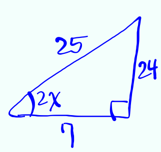
Find



$\sin 2x = 2\sin x\cos x$

$= 2\cdot\frac{-3}{5}\cdot\frac{-4}{5} = \boxed{\frac{24}{25}}$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\cdot\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{6}{4}}{1-\frac{9}{16}}$

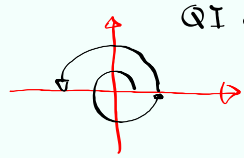


$= \frac{24}{16-9} = \boxed{\frac{24}{7}}$ $\quad$ LCD = 16

x is in QIII $\quad 180^\circ < x < 270^\circ$

What about $2x$? $\quad 360^\circ < 2x < 540^\circ$

QI or QII



$2x$ is in QI

Oct 7-11:02 AM

Find exact Value of $\cos\frac{\pi}{12}$

$\frac{\pi}{12}$ Rad $= \frac{180}{12}$ Degrees $= 15^\circ = 45^\circ - 30^\circ$

$\cos\frac{\pi}{12} = \cos 15^\circ = \cos(45^\circ - 30^\circ)$

$\cos(A-B) = ?$

$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$

Oct 7-11:11 AM

Find the exact value of

$\sin\frac{\pi}{12} + \sin\frac{5\pi}{12}$

$\frac{\pi}{12} = 15^\circ = 45^\circ - 30^\circ$

$\frac{5\pi}{12} = 75^\circ = 45^\circ + 30^\circ$

$\sin\frac{\pi}{12} + \sin\frac{5\pi}{12} = \sin(45^\circ - 30^\circ) + \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cos 30^\circ \cancel{- \cos 45^\circ \sin 30^\circ}$

$\sin 45^\circ \cos 30^\circ \cancel{+ \cos 45^\circ \sin 30^\circ}$

$= 2 \sin 45^\circ \cos 30^\circ = \cancel{2} \cdot \frac{\sqrt{2}}{\cancel{2}} \cdot \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{6}}{2}}$

Oct 7-11:15 AM

Verify

$$(\sin x + \cos x)^2 = 1 + \sin 2x$$

Recall $(A+B)^2 = A^2 + 2AB + B^2$

$$(\sin x + \cos x)^2 = \sin^2 x + \underbrace{2\sin x\cos x} + \cos^2 x$$

$$= \boxed{1 + \sin 2x}$$

Oct 7-11:20 AM

Verify

$$\frac{2\tan x}{1+\tan^2 x} = \sin 2x$$

$$\frac{2\tan x}{1+\tan^2 x} = \frac{\frac{2\sin x}{\cos x}}{1 + \frac{\sin^2 x}{\cos^2 x}} = \frac{\cos^2 x \cdot \frac{2\sin x}{\cancel{\cos x}}}{\underbrace{\cos^2 x + \sin^2 x}_{1}}$$

LCD = $\cos^2 x$

$$= \cos x \cdot 2\sin x$$

$$= 2\sin x\cos x$$

$$= \sin 2x$$

Oct 7-11:23 AM

Verify

Hint: $4x = 2 \cdot 2x$

$$\frac{\sin 4x}{\sin x} = \boxed{4}\cos x \boxed{\cos 2x}$$

$\sin 2A =$

$$\frac{\sin 4x}{\sin x} = \frac{\sin 2(2x)}{\sin x} = \frac{2\sin 2x \boxed{\cos 2x}}{\sin x}$$

$$= \frac{\boxed{2 \cdot 2}\cancel{\sin x}\cos x \cdot \boxed{\cos 2x}}{\cancel{\sin x}}$$

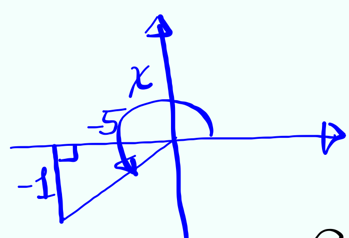
$$= 4\cos x \cos 2x$$

Oct 7-11:27 AM

$\cot x = 5 \quad 180^\circ < x < 270^\circ$, find $\cot 2x$.

Hint:

$\tan x = \frac{1}{5}$



$= \frac{12}{5}$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x} = \frac{2 \cdot \frac{1}{5}}{1 - \frac{1}{25}} = \frac{10}{25-1}$$

LCD = 25

$$= \frac{10}{24} = \frac{5}{12}$$

Oct 7-11:33 AM

Simplify

$A^2 - B^2 = (A+B)(A-B)$

$$\frac{2(\tan x - \cot x)}{\tan^2 x - \cot^2 x} = \frac{2\cancel{(\tan x - \cot x)}}{(\tan x + \cot x)\cancel{(\tan x - \cot x)}}$$

$$= \frac{2}{\tan x + \cot x} = \frac{2}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$$

LCD = $\sin x \cos x$

$$= \frac{2\sin x \cos x}{\sin^2 x + \cos^2 x} = \frac{\sin 2x}{1} = \boxed{\sin 2x}$$

Oct 7-11:37 AM